A trip down convexity lane:

Why investors should not overlook the effects of Convexity

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Investment Management

Introduction

A relatively calm period of low rates was ended by high inflation and central banks hiking interest rates to combat this. The quickly rising interest rates in 2022 and 2023 means that Convexity or the effect of Convexity is back in play. Not only for an investor seeking to outperform a benchmark, but also for pension funds or insurance companies seeking to hedge the interest rate risk of its liabilities. Their liabilities can run well into the next century, while their main hedging instrument (IRS) has a maximum maturity of 50 years. This means that they are short convexity even if they are fully hedged on paper. This highlights the need to look at the effects of Convexity.

Summary

- After a relatively calm period of low rates, volatility is back and so is Convexity!
- Convexity is a valuable feature and its effects become clear during periods of high volatility.
- There are several ways of setting up a Convexity trade, where a 30s50s curve is arguably the most pure trade in linear space.
- Convexity is not free of charge, there is a certain ‘give up’ cost associated with it.
- In the end being long or short Convexity should be an active decision!

In this paper we examine and explain the effects of Convexity. We start by dissecting the price-yield relationship and why there is need for a Convexity adjustment in it. We show the effects of Convexity for bonds and IRS and introduce a third-order effect called: “Speed”. Next we apply the established theoretical framework of Convexity in the real world and set up a Convexity trade. Finally we discuss the costs and risks associated with this trade and run a historical analysis on it.

Once upon a time in fixed-income space

Duration is probably the most intuitive and actively used metric by portfolio managers. Duration describes the price-yield relationship for fixed-income products, such as: bonds and interest rate derivatives. This relationship is often denoted as:

$$\%\Delta PV \approx -ModDur \times \Delta Yield \qquad (1)$$

The minus sign indicates an inverse relationship between yield and price; an upward shift in the yield will have a negative effect on the price, and vice versa. This is the (negative) linear first-order effect of the price-yield relationship and holds for small changes in the yield curve. However, in reality, the price-yield relationship is non-linear. This is why a second-order effect, called Convexity, is needed to properly explain the price-yield relationship. The impact of Convexity is often overlooked and becomes more visible with large yield curve shifts. It is usually added to equation (1) as a “Convexity adjustment” to the price-yield relationship:

$$\%\Delta PV \approx (-ModDur \times \Delta Yield) + [1/2 \times Convexity \times (\Delta Yield)^2] \qquad (2)$$

As mentioned before; duration is usually the most important consideration for portfolio management, because of ease of calculation and since it will capture most of the price change given a yield curve change, anyway. If that is the case, then when does Convexity matter? Because its impact can still be substantial, and being long or short Convexity should be an active decision.

Origin story

A good starting point is to ask the question: why does Convexity exist and what is the impact? In other words, why does the price-yield relationship has a convex shape? This is best illustrated by a simple example. Consider a 10y zero-coupon bond with a notional of €1,000 trading at par. It follows that duration is 10 and restating this duration in a Delta (or DV01) gives a Delta of 1. This means that for every 1 bps increase (decrease) in the yield curve, the price is expected to decrease (increase) by €1. This corresponds to equation 1 and the relationship is linear. Either way the loss or gain is equal in absolute terms: you gain €1 or lose €1 depending on the direction of the yield curve shift.

However we are omitting a very important factor: the time-value of money. The value of a bond is calculated by discounting it's cashflows. And when interest rates increase by 1 bp those cashflows are discounted at a higher interest rate, reducing the (present) value. If we assume a flat yield curve of 0% we can calculate the present value of the bond:

- 1000/(1.000)^10 = 1000

When interest rates change by 1 bp (up or down) the present value will be:
- Up: 1000/(1.0001)^10 = 999.00055
- Down: 1000/(0.9999)^10 = 1001.00055

In both the up or down move the value changes by about €1, like we would expect from the Delta approach. Though not exactly, the gain in a down move is slightly bigger than €1 and the loss in a up move is slightly smaller than €1. This effect is amplified when the change in interest rates increases, see table 1.

Table 1 Value bond after interest rate changes

	1 bp		100 bp		1000 bp	
	Delta	Delta + Convexity	Delta	Delta + Convexity	Delta	Delta + Convexity
Up	€ 999	€ 999	€ 900	€ 905	€ -	€ 386
Down	€ 1,001	€ 1,001	€ 1,100	€ 1,106	€ 2,000	€ 2,868

Source: Achmea IM

In the most extreme scenario (1,000bp interest rate change) the value of the bond doesn't drop to 0 when rates increase, but to € 386. When rated decrease the value of the bond doesn't double to €2,000, but increases even more to €2,868. It is clear that the linear or Delta approach, to calculate the value of a bond or the change in value is missing an important element, the Convexity adjustment from formula 2.
So the convex price-yield relationship stems from the fact that increases and decreases in the price are discounted at different yield curves. Furthermore, we can deduct from the simplified example of the 10y zero coupon bond, this Convexity is a valuable feature in favor of a bondholder.

Sequel

In the previous paragraphs we have explained the concept of Convexity and established a stylized concept for a long bond position. Let's take a deeper dive and apply this concept to the world of Interest Rate Swaps (IRS). A long position in an IRS, as for bonds, exhibits positive Convexity. Remember that a fixed-for-floating IRS is basically a levered bond position – for a receiver swap you are long a fixed-rate bond and short a floating-rate bond. In other words, being long an IRS – that is, receiving fixed and paying floating – is equivalent to a long bond position with leverage. As a consequence, the same concept of Convexity should apply to IRS.

Let's first look how Convexity behaves in relation to the maturity. We know that, all else equal, duration increases along the tenor-axis. The same goes for Convexity: longer dated swaps have, all else equal, more Convexity than shorter-dated swaps. So Convexity seems to be positively related to maturity. To illustrate this, let us consider three spot-starting par receiver swaps: 10y, 30y, and 50y:

Table 2 Convexity in IRS

	10y	**30y**	**50y**
Side	Receiver	Receiver	Receiver
Notional	€ 126,000,000	€ 52,000,000	€ 37,000,000
Coupon	Par	Par	Par
Delta	-100,647	-100,712	-100,610
Convexity	108	266	399

Source: Bloomberg, Achmea IM

Delta represents the change in the market value of the swap for a 1bp change in the yield curve. Note that it is negative and captures the negative first-order effect in the price-yield relationship (see equation 1) and is a measure of duration. Convexity is calculated as the rate of change of the Delta for a 1bps change in the yield curve. Note that it is positive and captures the positive second-order effect in the price-yield relationship (see equation 2). We can clearly see Convexity increasing with the tenor, given a constant Delta. As mentioned before, Convexity captures the non-linear part of the price-yield relationship. The non-linearity of Convexity makes sense if we look at a more detailed breakdown of the Delta in up and down moves:

Table 3 Impact of Convexity on Delta

	10y		30y		50y	
bp	Down	Up	Down	Up	Down	Up
1	-100,755	-100,540	-100,979	-100,446	-100,213	-101,008
50	-106,189	-95,413	-115,063	-88,282	-122,886	-82,759
100	-112,057	-90,469	-131,652	-77,502	-150,782	-68,403
200	-124,863	-81,379	-173,107	-59,999	-230,004	-47,425

Source: Bloomberg, Achmea IM

From the perspective of a receiver, as interest rates rise and the swap becomes more out of the money, your sensitivity to these rates declines. So with every rise in rates the market value of your swap declines less and vice versa for decreasing interest rates. We therefore can conclude that Convexity is nonlinear. From table 3 we can also conclude that long(er) dated swap provide additional Convexity compared to short(er) dated swaps.

Need for speed

We have now established that Delta is not constant and changes with interest rate shocks. This rate of change is non-linear and is measured by Convexity. The effects of Convexity, as shown before, can be quite significant for larger, but not unlikely, shocks. What about the rate of change of Convexity itself? The non-linearity implies that Convexity is not constant, but is this third-order effect noticeable? The rate of change of Convexity is called “Speed” or less catchy “DGammaDspot” and finds its origin in the famous “Greeks” that are extensively used in the options world. So there is a third-order effect, but is it still noticeable and significant enough to care about? In theory, these n-order effects are indefinitely, but become so small at some point that they are negligible. So do we stop looking after the second-order effect Convexity or should we look beyond and also care about third-order effects: the speed of a swap? We will use the 50y swap from above as an example to see the significance of third-order effects. See table below for a breakdown of the Convexity in up and down moves:

Table 4 Impact of speed on Convexity

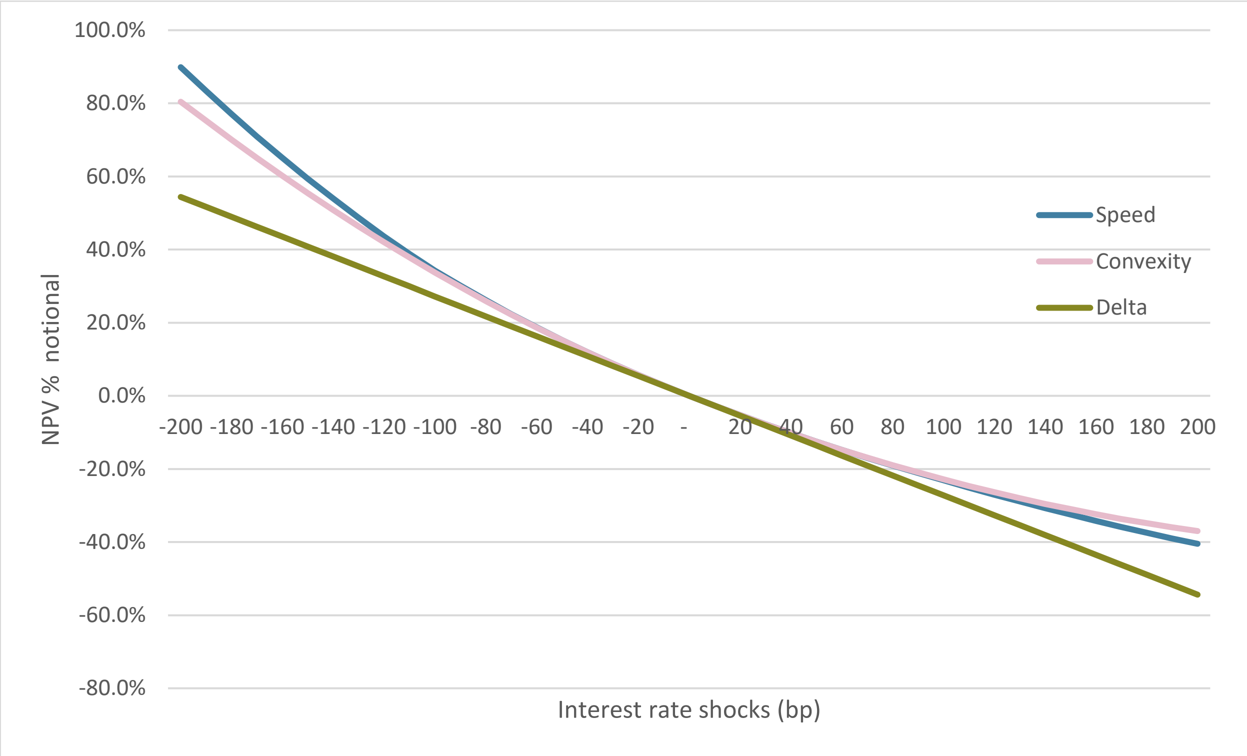
	50y	
bp	Down	Up
1	399	397
50	446	357
100	558	287
200	792	210

Source: Bloomberg, Achmea IM

The speed of Convexity is clearly noticeable; your sensitivity to Convexity almost doubles in a 200bp down move compared to a 1bp move. Granted, these seem to be extreme moves, however we have seen moves of 100bp within the span of just a few days not so long ago (e.g. UK LDI crisis, October 2022). The significance of

Convexity and Speed becomes more noticeable when we plot all the three order-effects: Delta, Convexity, and Speed.

Graph 1 The first 3 order-effects of the price-yield relationship



Source: Bloomberg, Achmea IM

Where the grey line only uses Delta to approximate the price-yield relationship, the orange line uses Delta plus Convexity and the blue line includes Speed as well. This gives an insightful visualization of the impact of every order-effect. It clearly shows, that Delta does a poor job at explaining the market value change for larger shocks. It also shows, quite surprisingly, for even larger shocks (but not unlikely) that Convexity alone does not get the job done as well. For larger shocks (e.g. >100bp) we do need to include the rate of change of Convexity: Speed[1].

So yes, we do have a need for speed when considering Convexity! Now that we have established the power behind Convexity and Speed we are going to look at real life applications aimed to benefit from these factors. In other words; how do you set up a Convexity trade?

The hitchhiker's guide to a Convexity trade

There are multiple ways of getting exposure to Convexity, but are there also ways to set up a pure Convexity trade? We will start with the bond market. We have established in previous paragraphs that Convexity is most apparent in longer tenors. When thinking of long-dated bonds; the notorious “schnitzels” immediately comes to mind. Conveniently the Austrian government have issued two so-called century bonds and even more conveniently from a Convexity analysis point of view; one with a high coupon and one with a low coupon. The RAGB 0.85 06/30/2120 yields a considerably higher Convexity than his younger brother RAGB 2.1 09/20/2117. Going long the 2120 and short the 2117, should give a long Convexity position; which can be Delta hedged. The main risk is a 94s97s curve. Obviously this is a very illiquid and specific curve, where there should be little noise

[1] Further n order-effects will most likely become significant as well if the shock is large enough. But the shocks become very unlikely and therefore we do not deem them relevant and are out of scope for this paper.

since markets most likely do not have a convincing view on this curve[2]. So the main difference between the two should be caused by Convexity. Below table shows a stylized example of a possible (outright) Delta-hedged Convexity trade:

Table 6 Scenario analysis Convexity trade Austrian government bonds ("Schnitzels")

	RAGB 2.1 09/20/2117		RAGB 0.85 06/30/2120		Net position	
Side	Short		Long			
Notional	€	43,000,000	€	58,100,000		
Coupon (%)		2.1		0.85		
Delta		99,929		-100,002		**0%**
Convexity		-609		731		**16.7%**
Initial outlay	€	(31,210,185)	€	23,428,663	**€**	**(7,781,522)**
Outright up	€	7,751,885	€	(7,413,316)	**€**	**338,569**
Outright down	€	(13,562,777)	€	14,340,904	**€**	**778,127**
No Move	€	(927,278)	€	581,906	**€**	**(345,372)**

Source: Bloomberg, Achmea IM

The effect of Convexity is clearly visible when we look at parallel shocks. In both up and down outright moves you have a gain! But it also shows that a move is necessary, because if rates remain unchanged you lose money due to the carry. A major downside of this trade is the initial outlay required that needs to be managed. Another drawback is also that to be able to short a bond requires the use of the repo market. After Delta hedging 16.7% of the Convexity remains, which is modest when we compare this to a Convexity trade using IRS.

As most convexity is lodged in the long-end we look at the 50y swap rate, the longest tenor that is still sufficiently liquid. Receiving fixed in 50y outright yields some decent Convexity! However this comes with an outright Delta position which muddles the Convexity effect. The effects of Convexity are still visible (see table 7) – you gain more in a down move than you lose in an up move – but not as clear. The outright position makes this more a Delta trade than a Convexity trade.

Table 7 Scenario analysis 50y Outright[3]

	Outright portfolio	
	50y	
Side	Receiver	
Notional	€	37,000,000
Coupon	Par	
Delta		-100,610
Convexity		399
Flattening	€	519,969
Steepening	€	(521,713)
Outright up	€	(8,334,009)
Outright down	€	12,382,105
No Move	€	879,120

Source: Bloomberg, Achmea IM

[2] For example, how does a central bank hike/cut, Job data and or inflation data affect the 94s97s curve? There is most likely no general conviction.

[3] The flattening and steepening scenarios are based on -10bp and 10bp, respectively. The outright up and down moves are 100bp large. In the "No Move" scenario we disregard the floating leg, given the uncertain nature and we do not have a view on the 6m forwards.

A more pure Convexity trade is a 30s50s flattener (e.g. pay 30y and receive 50y) in which the outright Delta is zero. In this trade the effects of Convexity are perfectly displayed by looking at a parallel move (see table 8); up or down, either way you win! Another possibility is a forward swap, such as a 30y20y (e.g. a 20y swap, 30y forward). The advantage of a forward is the absence of coupons/cashflows which makes the trade more "clean". The downside of this forward is the residual outright Delta that remains, which gives the same issues as with the 50y swap mentioned above; it muddles the pure Convexity effect.

Let's have a closer look at these two curve trades:

Table 8 Scenario analysis curve trades IRS

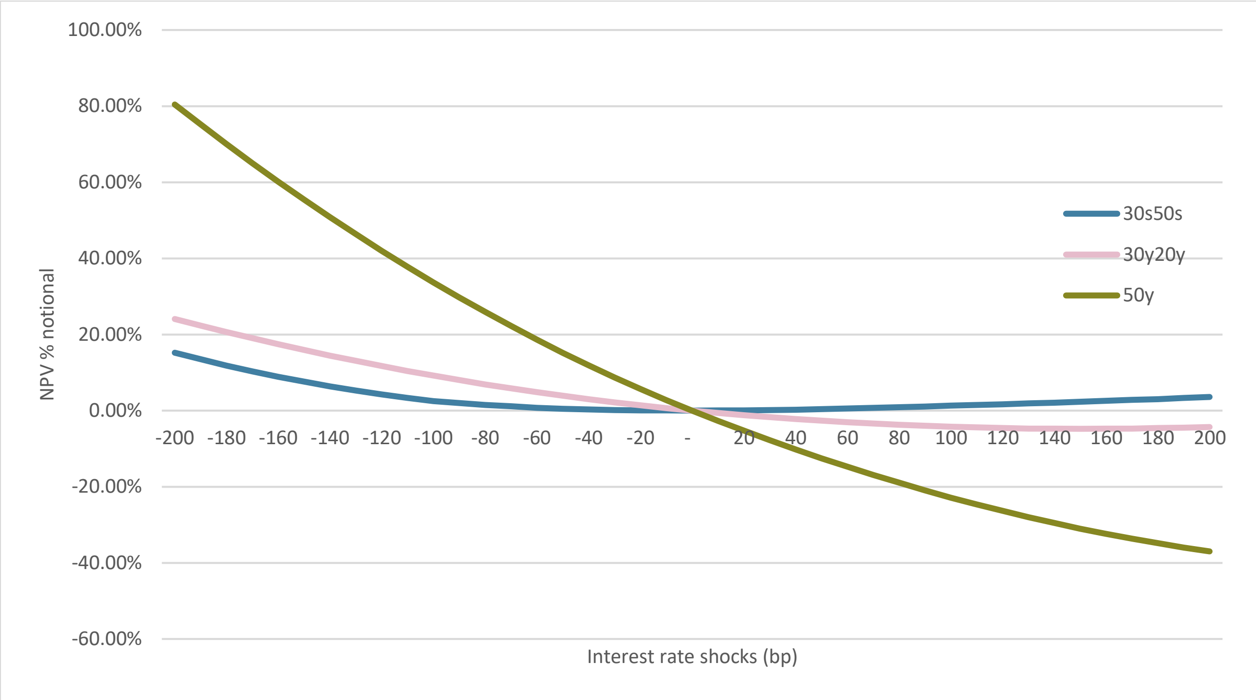
	30y	50y	Net	30y20y	Net
Side	Payer	Receiver		Receiver	
Notional	52,000,000.00	€ 37,000,000		€ 39,000,000	
Coupon	Par	Par		Par	
Delta	100,712	-100,610	**0%**	€ (24,753)	**24.60%**
Convexity	-266	399	**167%**	€ 193	**40.06%**
Flattening	€ 500,240	€ 519,969	**€ 1,020,209**	€ 880,257	**€ 880,257**
Steepening	€ (506,911)	€ (521,713)	**€ (1,028,624)**	€ (888,765)	**€ (888,765)**
Outright up	€ 8,855,677	€ (8,334,009)	**€ 521,668**	€ (1,690,460)	**€ (1,690,460)**
Outright down	€ (11,543,548)	€ 12,382,105	**€ 838,556**	€ 3,648,714	**€ 3,648,714**
No Move	€ (1,404,520)	€ 879,120	**€ (525,400)**	€ -	**€ -**

Source: Bloomberg, Achmea IM

The first thing to notice between the two portfolios is that the outright Delta is reduced to zero in the 30s50s portfolio, but this also decreases the Convexity to 33% of the original Convexity in a 50y swap. This makes sense, since there is Convexity in a 30y swap as well and in the curve portfolio we are short the 30y swap. In the 30y20y the residual Convexity is 40%, however the outright Delta is reduced to 25% of the original Delta in a 50y outright position instead of zero. Both portfolios benefit from a flattening in the 30s50s curve and similarly lose in a steepening move, we will come back to this later on.

The major difference comes to light when we look at the parallel moves and this is why the 30s50s trade is a better, or at least more pure, Convexity trade. If you are long the 30s50s, you always gain in a parallel move! It does not matter if rates go up or down; the position will yield a benefit. This is probably the greatest advocate for Convexity, and why it is such an appealing feature to investors. Plotting the three different Convexity trades gives a nice visualization of the Convexity effects and understates that the 30s50s spread arguably is the most pure Convexity trade.

Graph 3 Convexity trades



Source: Bloomberg, Achmea IM

There is no such thing as a free lunch

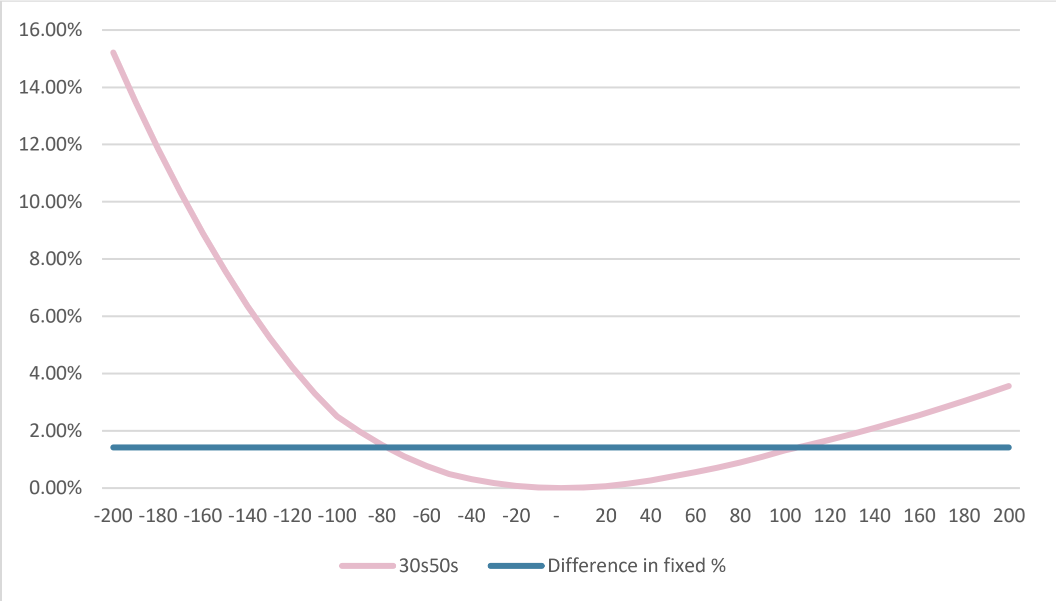
Before you run off to the swap store and buy yourself some decent amounts of 30s50s, it is not all sunshine and roses; there are costs and risks associated. Valuable features in – normally functioning – financial markets should always come at a price and they usually do. Since Convexity is such a valuable feature to investors it should come at a cost.

First of all, the "no move[4]" scenario above shows that you do need "a move" for Convexity to pay off. This scenario shows the (negative) monetary impact of the difference between the 30y rate and 50y rate. If the 30s50s spread is negative (30y>50Y) you give up this difference to get Convexity and vice versa you would, effectively, get paid for this feature. The flatter (more negative spread) the curve the more it "costs" you to get Convexity. Comparing this "give up cost" with the monetary amount gained from Convexity for a given interest rate shock, gives us a nice insight in the concept of Convexity, its asymmetric nature and answers the question: how much of an interest rate change do you need to make up for the difference between a 30y and 50y swap rate?

To find this "break-even point" we first translate the Convexity into a EUR amount gained from Convexity for a given interest rate shock. The difference, in monetary terms, between a 30y and 50y swap can be used as a measure of the value of additional Convexity. If we express this difference as a percentage of the notional we can compare the annual benefit from a certain interest rate shock to the difference in fixed rates between 30y and 50y, which is adjusted for the difference in notional (to maintain a Delta neutral comparison).

[4] Calculated as the difference between the fixed cash flows in both swaps, hence it excludes other possible factors like roll down.

Graph 4 Convexity value for parallel shifts vs. “give up cost”



Source: Bloomberg, Achmea IM

Table 9 Break-even point

Break even			
Bp move	Convexity value in EUR	Convexity value in % notional	Difference in fixed %
-81	525,400.00	1.42%	1.42%
112	525,400.00	1.42%	1.42%

Above graph and table show that we need a 81bp outright down move in the 50y fixed rate to precisely offset the difference in fixed rate between a 30y- and 50y swap. In an up move we need 112bp outright move to break even. Disregarding non-parallel (flattening/steepening) moves, this means that the swap curve should at least decrease by 81bp or increase by 112bp every year to make up for the 142bp you give up by being long 50y and short the 30y.

To get a bit of feeling of the likelihood of these moves, we can look at multiple historical periods and use the annualized volatility of the 50y rate in bp as a proxy for this likelihood. On average, historical annualized volatility in bp has been between 50bp-75bp. So being long Convexity most likely on average would not have covered the give up cost. However, these are considerably long periods, for shorter periods of time, it might very well pay-off to be long Convexity (remember extreme moves like during the UK LDI crisis).

Besides the give up cost there is a second cost associated with this trade: the risk of steepening. In above stylized analysis we are disregarding curvature changes[5], and only considering parallel moves to emphasize the effects and power of Convexity. Curvature changes, can however have a serious impact on a curve position (see tables 7&8). Steepening can easily wipe out the benefits from Convexity. In the intermezzo below we add everything together and apply above stylized analysis to a real life analysis.

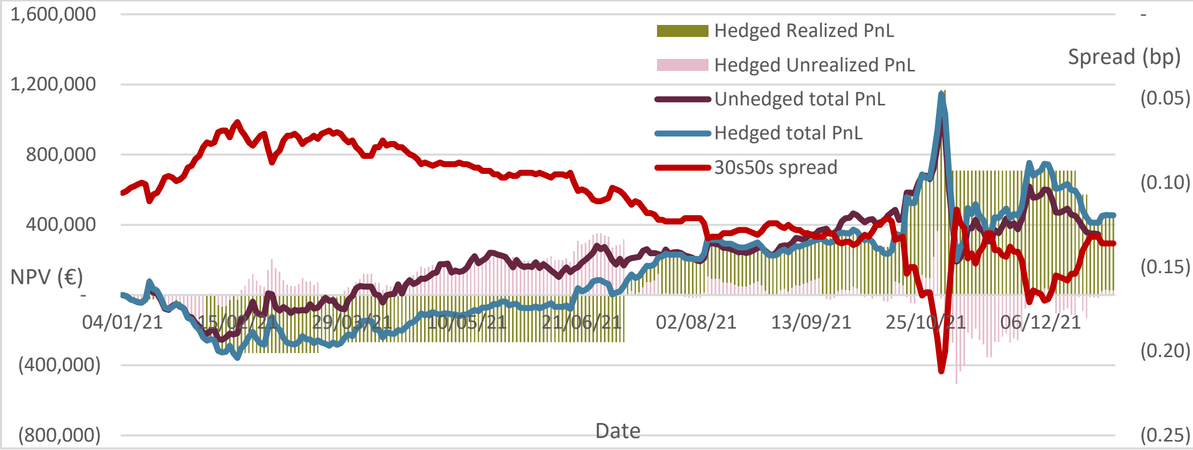
Intermezzo: real life Convexity trade

We set up a 30s50s using real life data – taking into account parallel, slope and curvature – and implement a trading strategy around it. There are multiple strategies to lock in profits and cut losses in the Convexity trade. Largely, one could choose between locking in small but more frequent profits or larger but less frequent profits.

[5] For ease of explaining the effects of convexity we created a scenario with parallel shocks.

Hedging prevents outright Delta to slip in and reduces risk and keeps it a clean Convexity trade. There are multiple ways of hedging, but the most clean way is to roll the 30s50s into a new spot starting 30s50s with the original curve Delta. We run both a hedged and an unhedged strategy for a period of one year[6].

Graph 5 hedged and unhedged Convexity strategies



The results clearly show that curvature changes have a large impact on a Convexity trade like a 30s50s spread. This is something to be warry off when setting up a Convexity trade. Furthermore we see the hedged strategy outperforming the unhedged strategy for the total period while reducing risk. However hedging does not always deliver a gain during the period.

We do have to conclude that the price or cost of Convexity in the end depends on one's view. A significant negative 30s50s spread might not sound so expensive if you expect the curve to flatten by a decent amount, and vice versa.

Endgame

Convexity is a really appealing feature to investors and its value becomes clear during periods of high volatility. In those periods of high volatility the benefits of Convexity are growing exponentially, underlying the significance of Convexity. There are several ways of setting up a Convexity trade, where a 30s50s spread is arguably the most pure play.

The assessment of the cost associated with holding a Convexity position against a uncertain return and the curve risk of the position is not clear cut. At current market conditions the 30s50s spread might look expensive, but is it expensive enough to be short Convexity? Clearly, this is also not that obvious. What is obvious, is that Convexity– although a second-order effect[7] – should definitely not be overlooked by investors and being long or short should be an active consideration. Especially Pension Funds and Insurers – given their natural short position – should be aware of the effects of Convexity.

So whether it concerns trade ideas, positioning or the entire portfolio; when thinking about Duration and Delta, investors should always keep in mind the effects of Convexity.

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[6] We have chosen 2021, given the extreme moves in 2022 and 2023. Furthermore we do not have a full year for 2023. We use bandwidths of 2.5%.

[7] Even third order-effects (e.g. Speed) appear to be quite noticeable and significant for large moves.

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